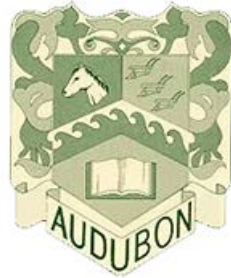


Audubon Public School District



Pre-Calculus

Curriculum Guide

Developed by:

Mr. Ron Latham

Ms. Lori Ann Tanenbaum

August 4, 2021

Table of Contents

Cover Page	Page 1
Table of Contents	Page 2
Course Description	Page 3
Overview / Progressions	Page 4
Unit 1	Page 10
Unit 2	Page 18
Unit 3	Page 24
Unit 4	Page 31
Unit 5	Page 39
Unit 6	Page 44
Appendix A: Previous Curriculum Documents	Page 50



Course Description

Pre-Calculus

As preparation for Calculus this course will involve a formal study of trigonometry, analytic geometry and advanced algebra. The study of trigonometry is based on the knowledge of elementary algebra and geometry. Students will briefly re-examine some properties of the set of real numbers and study in detail circular functions and complex numbers with graphic and algebraic solutions and applications. The study of analytic geometry will be based upon an understanding of the basic principles of algebra as they apply to analytic geometry. Students will study coordinate lines and planes, equations and graphics, circles, conic sections, transformations of functions, graphs of equations of higher degree, polar coordinates and parametric equations. The study of advanced algebra provides a rich preparation for college courses in calculus, abstract algebra and analytical geometry. Other topics of study will include statements and sets in mathematics, ordered fields, mathematical induction, functions, graphs of polynomial functions and exponential and logarithmic functions, their graphs and applications.

Overview / Progressions

Overview	Standards for Mathematical Content	Unit Focus	Standards for Mathematical Practice
<p style="text-align: center;">Unit 1 Basic Functions & Graphs</p>	<ul style="list-style-type: none"> ● F-IF:B.4 ● F-IF:C.7 ● F-BF:A.1 ● F-BF:B.4 	<ul style="list-style-type: none"> ● Model and solve equations and applications. ● Analyze function characteristics, properties of functions, and the 12 basic function graphs. ● Build functions from functions using composition. ● Define graphical transformations and perform transformations of the 12 basic functions. 	<p>MP.1 Make sense of problems and persevere in solving them.</p> <p>MP.2 Reason abstractly and quantitatively.</p> <p>MP.3 Construct viable arguments & critique the reasoning. of others.</p>
<p style="text-align: center;">Unit 2 Quadratic, Power, Polynomial & Rational Functions</p>	<ul style="list-style-type: none"> ● N-CN:A.1-3 ● N-CN:C.7-9 ● A-REI:B.4 ● F-IF:C.7.c-d 	<ul style="list-style-type: none"> ● Analyze quadratic functions and their characteristics through graphs and applications. ● Analyze power functions and sketch their graphs. ● Graph polynomial functions, predict end behavior, and find the 	<p>MP.4 Model with mathematics.</p>

		<p>real zeros.</p> <ul style="list-style-type: none"> • Divide polynomials using both long division and synthetic division. Apply the Remainder Theorem, Factor Theorem, and Rational Zeros Theorem. • Add, subtract, multiply and divide complex numbers. Find the complex zeros of quadratic functions. • Describe the graphs of rational functions: identifying horizontal and vertical asymptotes and predict the end behavior of the graphs. • Solve equations and inequalities in one-variable using algebraic and graphic techniques. 	<p>MP.5 Use appropriate tools strategically..</p> <p>MP.6 Attend to precision.</p> <p>MP.7 Look for and make use of structure.</p> <p>MP.8 Look for and express regularity in repeated reasoning.</p>
<p>Unit 3 Exponential, Logistic, and Logarithmic Functions</p>	<ul style="list-style-type: none"> • A-SSE:B.3c • F.IF:C.7e • F.IF:C.8.a-b • F-BF:B.5 	<ul style="list-style-type: none"> • Evaluate exponential expressions. • Identify and graph exponential and logistic functions. 	

		<ul style="list-style-type: none"> ● Use exponential growth, decay, and regression to model real-life applications. ● Evaluate and graph common and natural logarithms. ● Apply the properties of logarithms to evaluate expressions and graph functions. ● Apply the properties of logarithms to solve exponential and logarithmic equations, algebraically. 	
<p style="text-align: center;">Unit 4 Trigonometric Functions</p>	<ul style="list-style-type: none"> ● F-TF:A.1-4 ● F-TF:B.5-7 ● G-C:B.5 ● G-SRT: C.6,8 	<ul style="list-style-type: none"> ● Evaluate and graph trigonometric functions and their inverses. ● Convert between radian and degree measurement. ● Develop the unit circle and evaluate trigonometric functions using the unit circle. ● Sketch graphs of trigonometric functions and define sinusoidal 	

		<p>characteristics.</p> <ul style="list-style-type: none"> ● Use trigonometric ratios to solve real-world applications, including mechanics and navigation. 	
<p>Unit 5 Analytic Trigonometry</p>	<ul style="list-style-type: none"> ● F-TF:B.5-7 ● F-TF:C.8-9 	<ul style="list-style-type: none"> ● Develop and define trigonometric identities. ● Use trigonometric identities to simplify expressions and evaluate equations. ● Use inverse trigonometric functions to solve trigonometric equations. ● Use multiple-angles and half-angle formulas to rewrite and evaluate trigonometric functions. 	
<p>Unit 6 Introduction to Calculus</p>	<ul style="list-style-type: none"> ● F-BF:A.1a,c ● F-BF:B.4d ● F-TF:B.7 	<ul style="list-style-type: none"> ● Introduce and define the concept of limit. ● Use the properties of limits to evaluate limits. ● Evaluate one-sided limits. 	

		<ul style="list-style-type: none">● Evaluate limits of functions at infinity.● Use differentiation definitions to evaluate the derivative of functions.	
--	--	--	--

Subject: Pre-Calculus	Grade: 11-12	Unit: 1 Basic Functions & Graphs	1st Marking Period
Content Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills	
<ul style="list-style-type: none"> ● F-IF:B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. <i>Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*</i> 	<p>MP.1 Make sense of problems and persevere in solving them.</p> <p>MP.2 Reason abstractly and quantitatively.</p> <p>MP.3 Construct viable arguments & critique the reasoning. of others.</p> <p>MP.4 Model with mathematics.</p> <p>MP.5 Use appropriate tools strategically.</p> <p>MP.6 Attend to precision.</p> <p>MP.7 Look for and make use of structure.</p> <p>MP.8 Look for and express regularity in repeated</p>	<p>Concept(s):</p> <ul style="list-style-type: none"> ● The twelve basic functions and their properties <p>Students are able to:</p> <ul style="list-style-type: none"> ● find the domain and range of functions. ● determine the intervals in which the functions are increasing, decreasing, or constant. ● determine the relative extrema of the functions. ● recognize the graphs of parent functions <p style="text-align: center;">Learning Goal 1: Identify the characteristics of functions and use the knowledge of the characteristics to graph the functions, by hand.</p>	

	reasoning.	
<ul style="list-style-type: none"> ● F-IF:C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are 	<p>MP.1 Make sense of problems and persevere in solving them.</p> <p>MP.2 Reason abstractly and quantitatively.</p> <p>MP.3 Construct viable arguments & critique the reasoning. of others.</p> <p>MP.4 Model with mathematics.</p> <p>MP.5 Use appropriate tools strategically.</p> <p>MP.6 Attend to precision.</p> <p>MP.7 Look for and make use of structure.</p> <p>MP.8 Look for and express regularity in repeated reasoning.</p>	<p>Concept(s):</p> <ul style="list-style-type: none"> ● Continuation of the basic functions and their properties <p>Students are able to:</p> <ul style="list-style-type: none"> ● graph additional parent functions without a calculator. ● interpret and graph vertical and horizontal shifts. ● identify piecewise-defined functions. ● graph piecewise functions. <p>Learning Goal 2: Expand knowledge of Learning Goal 1 to include additional graphing characteristics and piecewise-defined functions.</p>

<p>available, and showing end behavior.</p> <p>e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.</p>		
<p>● F-BF:A.1</p> <p>Write a function that describes a relationship between two quantities.*</p> <p>a. Determine an explicit expression, a recursive process, or steps for calculation from a context.</p> <p>b. Combine standard function types using arithmetic operations.</p> <p><i>For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.</i></p>	<p>MP.1 Make sense of problems and persevere in solving them.</p> <p>MP.2 Reason abstractly and quantitatively.</p> <p>MP.3 Construct viable arguments & critique the reasoning. of others.</p> <p>MP.4 Model with mathematics.</p> <p>MP.5 Use appropriate tools strategically.</p> <p>MP.6 Attend to precision.</p> <p>MP.7 Look for and make use of structure.</p>	<p>Concept(s): Building functions from functions</p> <p>Students are able to:</p> <ul style="list-style-type: none"> ● add, subtract, multiply, and divide functions. ● find compositions of one function with another function. ● use a combination of functions to model and solve real-life applications. <p>Learning Goal 3: Combine functions algebraically and use knowledge of function characteristics to analyze the composition of functions.</p>

<p>c. (+) Compose functions. <i>For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.</i></p>	<p>MP.8 Look for and express regularity in repeated reasoning.</p>	
<p>● F-BF:B.4 Find inverse functions. a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. <i>For example, $f(x) = 2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$.</i> b. (+) Verify by composition that one function is the inverse of another.</p>	<p>MP.1 Make sense of problems and persevere in solving them. MP.2 Reason abstractly and quantitatively. MP.3 Construct viable arguments & critique the reasoning of others. MP.4 Model with mathematics. MP.5 Use appropriate tools strategically.</p>	<p>Concept(s):</p> <ul style="list-style-type: none"> ● Inverse relations and inverse functions <p>Students are able to:</p> <ul style="list-style-type: none"> ● compute inverses of functions. ● verify that two functions are inverse functions of each other ● find inverse functions algebraically <p>Learning Goal 4: Given a function, find the formula for inverse function using algebraic techniques.</p>

<p>c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.</p> <p>d. (+) Produce an invertible function from a non-invertible function by restricting the domain.</p>	<p>MP.6 Attend to precision.</p> <p>MP.7 Look for and make use of structure.</p> <p>MP.8 Look for and express regularity in repeated reasoning.</p>	
--	---	--

Formative Assessments	Summative Assessments
<ul style="list-style-type: none"> ● Student Whiteboards ● Checks for Understanding ● Teacher’s observation ● Desmos Activities ● Section Quizzes ● IXL 	<ul style="list-style-type: none"> ● Chapter Test ● Midterm ● Common Assessment ● Benchmark Assessment
Suggested Primary Resources	Suggested Supplemental Resources
<p>PRECALCULUS - Functions & Graphs: 4th Edition TI-83/84 (College Prep) TI-89 (Honors) Desmos Graphing Calculator</p>	<p>IXL Desmos Activities Edpuzzle Quizlet Khan Academy Tutorials</p>
Cross-Curricular Connections & 21st Century Skills	
<ul style="list-style-type: none"> ● Science, Technology, Engineering, and Mathematics (STEM) Literacy ● Critical Thinking & Problem Solving 	

- Communication and Collaboration
- Life and Career Skills

Essential Questions

- What are the 10 basic functions?
- How can the domain and range of a function be found graphically? Using algebra?
- What is asymptotic behavior, and how can asymptotes be found graphically and algebraically?
- How can algebraic tools like factoring, distributing, and simplifying rational expressions be used to analyze the behavior of functions?
- What is end behavior, and how is it related to the limit at infinity?
- How can coefficients and constants be used to translate, stretch, and rotate the graphs of functions?
- What are the limitations of graphing calculators, and what kind of functions often yield misleading results when solved technologically?
- How can curve-fitting technology be used to convert data into mathematical models, which can then be used to extrapolate future results?
- How can we verify by composition that one function is the inverse of another?

Enduring Understanding

- All functions can be categorized and compared in terms of their domain, range, inverse, boundedness, extrema, continuity, graph, intercepts and asymptotes.
- There are 10 major "families" of functions. Each family has its own shape when graphed, and each family has its own restrictions and limits.

Differentiation & Real World Connections

504	<ul style="list-style-type: none"> ● preferential seating ● extended time on tests and assignments ● reduced homework or classwork ● verbal, visual, or technology aids 	<ul style="list-style-type: none"> ● modified textbooks or audio-video materials ● behavior management support ● adjusted class schedules or grading ● verbal testing
Enrichment	<ul style="list-style-type: none"> ● Utilize collaborative media tools ● Provide differentiated feedback ● Opportunities for reflection ● Opportunities for self-evaluation 	<ul style="list-style-type: none"> ● Encourage student voice and input ● Model close reading ● Distinguish long term and short term goals
IEP	<ul style="list-style-type: none"> ● Utilize “skeleton notes” where some required information is already filled in for the student ● Provide access to a variety of tools for responses ● Provide opportunities to build familiarity and to practice with multiple media tools ● Graphic organizers 	<ul style="list-style-type: none"> ● Leveled text and activities that adapt as students build skills ● Provide multiple means of action and expression ● Consider learning styles and interests ● Provide differentiated mentors
ELLs	<ul style="list-style-type: none"> ● Pre-teach new vocabulary and meaning of symbols ● Embed glossaries or definitions ● Provide translations ● Connect new vocabulary to background knowledge 	<ul style="list-style-type: none"> ● Provide flash cards ● Incorporate as many learning senses as possible ● Portray structure, relationships, and associations through concept webs ● Graphic organizers
At-risk	<ul style="list-style-type: none"> ● Purposeful seating ● Counselor involvement ● Parent involvement 	<ul style="list-style-type: none"> ● Contracts ● Alternate assessments ● Hands-on learning

21st Century Skills

- Creativity
- Innovation
- Critical Thinking

- Problem Solving
- Communication
- Collaboration

Integrating Technology

- Chromebooks
- Internet research
- Online programs

- Virtual collaboration and projects
- Presentations using presentation hardware and software

Career education

- Weekly Discussions: The value of mastering multiple languages in the workforce.

- Equity Discussions: People who benefit from knowing multiple languages.

Subject: Pre-Calculus	Grade: 11-12	Unit: 2 Quadratic, Power, Polynomial & Rational Functions	1st & 2nd Marking Period
Content Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills	
<ul style="list-style-type: none"> ● N-CN:A.1-3 Perform arithmetic operations with complex numbers. 1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real. 2. Use the relation $i^2 = -1$, and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. 	<p>MP.1 Make sense of problems and persevere in solving them.</p> <p>MP.2 Reason abstractly and quantitatively.</p> <p>MP.3 Construct viable arguments & critique the reasoning of others.</p> <p>MP.4 Model with mathematics.</p> <p>MP.5 Use appropriate tools strategically.</p>	<p>Concept(s):</p> <ul style="list-style-type: none"> ● The zeros of polynomials are complex numbers. <p>Students are able to:</p> <ul style="list-style-type: none"> ● use the imaginary unit i to write complex numbers ● add, subtract, multiply and divide complex numbers ● use complex conjugates to write the quotient of two complex numbers in standard form <p style="text-align: center;">Learning Goal 1: Analyze polynomial functions and find the complex zeros of polynomial functions.</p>	

<p>3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.</p>	<p>MP.6 Attend to precision.</p> <p>MP.7 Look for and make use of structure.</p> <p>MP.8 Look for and express regularity in repeated reasoning.</p>	
<p>● N-CN:C.7-9</p> <p>Use complex numbers in polynomial identities and equations.</p> <p>7. Solve quadratic equations with real coefficients that have complex solutions.</p> <p>8. (+) Extend polynomial identities to the complex numbers. <i>For example, rewrite</i> $\square^2 + 4as(x + 2i)(x - 2i)$.</p> <p>9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.</p>	<p>MP.1 Make sense of problems and persevere in solving them.</p> <p>MP.2 Reason abstractly and quantitatively.</p> <p>MP.3 Construct viable arguments & critique the reasoning. of others.</p> <p>MP.4 Model with mathematics.</p> <p>MP.5 Use appropriate tools strategically.</p> <p>MP.6 Attend to precision.</p> <p>MP.7 Look for and make use of structure.</p> <p>MP.8 Look for and express regularity in repeated</p>	<p>Concept(s):</p> <ul style="list-style-type: none"> • The zeros of polynomials are complex numbers, continued. <p>Students are able to:</p> <ul style="list-style-type: none"> • factor polynomial functions with real coefficients • write a polynomial function given real and complex zeros • Find complex zeros using conjugates <p>Learning Goal 2: Create polynomial functions with real and complex zeros.</p> <p>Learning Goal 3: Factor polynomial functions with real and complex zeros.</p>

	reasoning.	
<ul style="list-style-type: none"> ● A-REI:B.4 <p>Solve quadratic equations in one variable.</p> <p>a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.</p>	<p>MP.1 Make sense of problems and persevere in solving them.</p> <p>MP.2 Reason abstractly and quantitatively.</p> <p>MP.3 Construct viable arguments & critique the reasoning. of others.</p> <p>MP.4 Model with mathematics.</p> <p>MP.5 Use appropriate tools strategically.</p> <p>MP.6 Attend to precision.</p> <p>MP.7 Look for and make use of structure.</p> <p>MP.8 Look for and express regularity in repeated reasoning.</p>	<p>Concept(s): Quadratic functions and modeling applications</p> <p>Students are able to:</p> <ul style="list-style-type: none"> ● evaluate transformations of quadratic functions. ● factor quadratic functions using the completing the square technique. ● factor quadratic functions using the quadratic formula. ● solve quadratic equations using multiple factoring techniques. <p style="text-align: center;">Learning Goal 4: Solving quadratic equations using mastery of factoring</p>

<ul style="list-style-type: none"> ● F-IF:C.7.c-d <p>Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. *</p> <p>c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.</p> <p>d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.</p>	<p>MP.1 Make sense of problems and persevere in solving them.</p> <p>MP.2 Reason abstractly and quantitatively.</p> <p>MP.3 Construct viable arguments & critique the reasoning. of others.</p> <p>MP.4 Model with mathematics.</p> <p>MP.5 Use appropriate tools strategically.</p> <p>MP.6 Attend to precision.</p> <p>MP.7 Look for and make use of structure.</p> <p>MP.8 Look for and express regularity in repeated reasoning.</p>	<p>Concept(s):</p> <ul style="list-style-type: none"> ● Polynomial functions of higher degree and graphing polynomial functions. <p>Students are able to:</p> <ul style="list-style-type: none"> ● analyze graphs of polynomial functions to predict degree and leading coefficients. ● predict the end behavior of a polynomial function. ● find the zeros of a polynomial function algebraically. <p>Learning Goal 5: Graph polynomial functions using function analysis to define function characteristics.</p>
--	--	--

504	<ul style="list-style-type: none"> ● preferential seating ● extended time on tests and assignments ● reduced homework or classwork ● verbal, visual, or technology aids 	<ul style="list-style-type: none"> ● modified textbooks or audio-video materials ● behavior management support ● adjusted class schedules or grading ● verbal testing
-----	---	---

Enrichment	<ul style="list-style-type: none"> ● Utilize collaborative media tools ● Provide differentiated feedback ● Opportunities for reflection ● Opportunities for self-evaluation 	<ul style="list-style-type: none"> ● Encourage student voice and input ● Model close reading ● Distinguish long term and short term goals
IEP	<ul style="list-style-type: none"> ● Utilize “skeleton notes” where some required information is already filled in for the student ● Provide access to a variety of tools for responses ● Provide opportunities to build familiarity and to practice with multiple media tools ● Graphic organizers 	<ul style="list-style-type: none"> ● Leveled text and activities that adapt as students build skills ● Provide multiple means of action and expression ● Consider learning styles and interests ● Provide differentiated mentors
ELLs	<ul style="list-style-type: none"> ● Pre-teach new vocabulary and meaning of symbols ● Embed glossaries or definitions ● Provide translations ● Connect new vocabulary to background knowledge 	<ul style="list-style-type: none"> ● Provide flash cards ● Incorporate as many learning senses as possible ● Portray structure, relationships, and associations through concept webs ● Graphic organizers
At-risk	<ul style="list-style-type: none"> ● Purposeful seating ● Counselor involvement ● Parent involvement 	<ul style="list-style-type: none"> ● Contracts ● Alternate assessments ● Hands-on learning
21st Century Skills		
<ul style="list-style-type: none"> ● Creativity ● Innovation ● Critical Thinking 	<ul style="list-style-type: none"> ● Problem Solving ● Communication ● Collaboration 	

Integrating Technology	
<ul style="list-style-type: none"> ● Chromebooks ● Internet research ● Online programs 	<ul style="list-style-type: none"> ● Virtual collaboration and projects ● Presentations using presentation hardware and software
Career education	
<ul style="list-style-type: none"> ● Weekly Discussions: The value of mastering multiple languages in the workforce. 	<ul style="list-style-type: none"> ● Equity Discussions: People who benefit from knowing multiple languages.

Subject: Pre-Calculus	Grade: 11-12	Unit: 3 Exponential, Logistic, and Logarithmic Functions	2nd & 3rd Marking Period
Content Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills	
<ul style="list-style-type: none"> ● A-SSE:B.3c <p>Use the properties of exponents to transform expressions for exponential functions. <i>For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.</i></p>	<p>MP.1 Make sense of problems and persevere in solving them.</p> <p>MP.2 Reason abstractly and quantitatively.</p> <p>MP.3 Construct viable arguments & critique the reasoning. of others.</p> <p>MP.4 Model with mathematics.</p> <p>MP.5 Use appropriate tools strategically.</p>	<p>Concept(s):</p> <ul style="list-style-type: none"> ● The transformations of exponential functions. <p>Students are able to:</p> <ul style="list-style-type: none"> ● identify the properties of exponential functions ● determine the transformations of exponential functions using the properties. ● compute exponential function values for rational number inputs ● graph logistic growth functions <p style="text-align: center;">Learning Goal 1: Evaluate exponential expressions and identify and graph exponential and logistic functions.</p>	

	<p>MP.6 Attend to precision.</p> <p>MP.7 Look for and make use of structure.</p> <p>MP.8 Look for and express regularity in repeated reasoning.</p>	
<ul style="list-style-type: none"> ● F.IF:C.7e <p>Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.</p>	<p>MP.1 Make sense of problems and persevere in solving them.</p> <p>MP.2 Reason abstractly and quantitatively.</p> <p>MP.3 Construct viable arguments & critique the reasoning of others.</p> <p>MP.4 Model with mathematics.</p> <p>MP.5 Use appropriate tools strategically.</p> <p>MP.6 Attend to precision.</p> <p>MP.7 Look for and make use of structure.</p> <p>MP.8 Look for and express regularity in repeated</p>	<p>Concept(s):</p> <ul style="list-style-type: none"> ● Exponential and logarithmic functions and their graphs <p>Students are able to:</p> <ul style="list-style-type: none"> ● graph the exponential and logarithmic parent functions. ● analyze the transformations of exponential and logarithmic functions. ● use exponential and logarithmic functions to model and solve real-world applications. <p>Learning Goal 2: Analyze exponential and logarithmic function characteristics and graph the functions, by hand.</p>

	reasoning.	
<p>● F.IF:C.8.a-b</p> <p>Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.</p> <p>a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.</p> <p>b. Use the properties of exponents to interpret expressions for exponential functions. <i>For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as</i></p>	<p>MP.1 Make sense of problems and persevere in solving them.</p> <p>MP.2 Reason abstractly and quantitatively.</p> <p>MP.3 Construct viable arguments & critique the reasoning. of others.</p> <p>MP.4 Model with mathematics.</p> <p>MP.5 Use appropriate tools strategically.</p> <p>MP.6 Attend to precision.</p> <p>MP.7 Look for and make use of structure.</p> <p>MP.8 Look for and express regularity in repeated reasoning.</p>	<p>Concept(s): Apply the properties of exponential and logarithmic functions.</p> <p>Students are able to:</p> <ul style="list-style-type: none"> ● convert exponential and logarithmic functions. ● evaluate exponential, common and natural logarithms using the properties of functions. ● expand and condense exponential and logarithmic expressions. <p style="text-align: center;">Learning Goal 3: Expand on the properties of functions by using the properties of logarithms and exponents to manipulate expressions and solve equations.</p>

<p><i>representing exponential growth or decay.</i></p>		
<ul style="list-style-type: none"> ● F-BF:B.5 <p>(+) Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents.</p>	<p>MP.1 Make sense of problems and persevere in solving them.</p> <p>MP.2 Reason abstractly and quantitatively.</p> <p>MP.3 Construct viable arguments & critique the reasoning. of others.</p> <p>MP.4 Model with mathematics.</p> <p>MP.5 Use appropriate tools strategically.</p> <p>MP.6 Attend to precision.</p> <p>MP.7 Look for and make use of structure.</p> <p>MP.8 Look for and express regularity in repeated reasoning.</p>	<p>Concept(s):</p> <ul style="list-style-type: none"> ● Logarithmic functions are the inverses of exponential functions <p>Students are able to:</p> <ul style="list-style-type: none"> ● use domains to identify the inverse relationships of exponential and logarithmic functions ● solve exponential and logarithmic equations using the properties of each function and the inverse relationship <p style="text-align: center;">Learning Goal 4: Understand the inverse relationship between exponents and logarithms to solve exponential and logarithmic problems.</p>

Formative Assessments	Summative Assessments
<ul style="list-style-type: none"> ● Student Whiteboards ● Checks for Understanding ● Teacher’s observation ● Desmos Activities ● Section Quizzes ● IXL 	<ul style="list-style-type: none"> ● Chapter Test ● Midterm ● Common Assessment ● Benchmark Assessment
Suggested Primary Resources	Suggested Supplemental Resources
PRECALCULUS - Functions & Graphs: 4th Edition TI-83/84 (College Prep) TI-89 (Honors) Desmos Graphing Calculator	IXL Desmos Activities Edpuzzle Quizlet Khan Academy Tutorials
Cross-Curricular Connections & 21st Century Skills	
<ul style="list-style-type: none"> ● Science, Technology, Engineering, and Mathematics (STEM) Literacy ● Critical Thinking & Problem Solving ● Communication and Collaboration ● Life and Career Skills 	
Essential Questions	Enduring Understanding
<ul style="list-style-type: none"> ● How are exponential and logarithmic functions related? 	<ul style="list-style-type: none"> ● Logarithms are another form of exponential function that can be solved following its own set of properties. ● \ln and e are inverse exponential functions of each other following their own set of rules for solution.

<ul style="list-style-type: none"> ● What are the shapes and properties of exponential and logarithmic functions, and how can they be transformed? ● How can populations, annuities, and radioactive half lives be modeled exponentially? ● How can the properties of exponential and logarithmic functions be used to manipulate and simplify problems? ● What are orders of magnitude and how are they related to exponential functions? ● How can $A = Pert$ and other formulas be used to model exponential growth and decay? ● How can the graphs of exponential and logarithmic functions show intercepts and end behavior? 	<ul style="list-style-type: none"> ● Logarithms can be solved by rewriting them as exponents, and vice-versa. ● Exponential formulas like: $A = Pert$ can be used to solve growth problems in science and business. ● The number e is used to model continuous growth and decay.
--	---

Differentiation & Real World Connections		
504	<ul style="list-style-type: none"> ● preferential seating ● extended time on tests and assignments ● reduced homework or classwork ● verbal, visual, or technology aids 	<ul style="list-style-type: none"> ● modified textbooks or audio-video materials ● behavior management support ● adjusted class schedules or grading ● verbal testing
Enrichment	<ul style="list-style-type: none"> ● Utilize collaborative media tools ● Provide differentiated feedback ● Opportunities for reflection ● Opportunities for self-evaluation 	<ul style="list-style-type: none"> ● Encourage student voice and input ● Model close reading ● Distinguish long term and short term goals

IEP	<ul style="list-style-type: none"> ● Utilize “skeleton notes” where some required information is already filled in for the student ● Provide access to a variety of tools for responses ● Provide opportunities to build familiarity and to practice with multiple media tools ● Graphic organizers 	<ul style="list-style-type: none"> ● Leveled text and activities that adapt as students build skills ● Provide multiple means of action and expression ● Consider learning styles and interests ● Provide differentiated mentors
ELLs	<ul style="list-style-type: none"> ● Pre-teach new vocabulary and meaning of symbols ● Embed glossaries or definitions ● Provide translations ● Connect new vocabulary to background knowledge 	<ul style="list-style-type: none"> ● Provide flash cards ● Incorporate as many learning senses as possible ● Portray structure, relationships, and associations through concept webs ● Graphic organizers
At-risk	<ul style="list-style-type: none"> ● Purposeful seating ● Counselor involvement ● Parent involvement 	<ul style="list-style-type: none"> ● Contracts ● Alternate assessments ● Hands-on learning
21st Century Skills		
<ul style="list-style-type: none"> ● Creativity ● Innovation ● Critical Thinking 	<ul style="list-style-type: none"> ● Problem Solving ● Communication ● Collaboration 	
Integrating Technology		

<ul style="list-style-type: none"> ● Chromebooks ● Internet research ● Online programs 	<ul style="list-style-type: none"> ● Virtual collaboration and projects ● Presentations using presentation hardware and software
Career education	
<ul style="list-style-type: none"> ● Weekly Discussions: The value of mastering multiple languages in the workforce. 	<ul style="list-style-type: none"> ● Equity Discussions: People who benefit from knowing multiple languages.

Subject: Pre-Calculus	Grade: 11-12	Unit: 4 Trigonometric Functions	3rd Marking Period
Content Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills	
<ul style="list-style-type: none"> ● F-TF:A.1-4 <p>Extend the domain of trigonometric functions using the unit circle</p> <p>1. Understand radian measure of an angle as the length of the arc on the unit</p>	<p>MP.1 Make sense of problems and persevere in solving them.</p> <p>MP.2 Reason abstractly and quantitatively.</p>	<p>Concept(s):</p> <ul style="list-style-type: none"> ● Trigonometric functions and the Unit Circle <p>Students are able to:</p> <ul style="list-style-type: none"> ● convert between radian and degree. ● define the six trigonometric functions using right triangle trigonometry. ● find coterminal angles. ● evaluate trigonometric functions using the unit circle. 	

<p>circle subtended by the angle.</p> <p>2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.</p> <p>3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for πx, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number.</p> <p>4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.</p>	<p>MP.3 Construct viable arguments & critique the reasoning. of others.</p> <p>MP.4 Model with mathematics.</p> <p>MP.5 Use appropriate tools strategically.</p> <p>MP.6 Attend to precision.</p> <p>MP.7 Look for and make use of structure.</p> <p>MP.8 Look for and express regularity in repeated reasoning.</p>	<p>Learning Goal 1: Use right triangle trigonometry to build the Unit Circle.</p> <p>Learning Goal 2: Use the Unit Circle to evaluate trigonometric functions.</p>
--	--	--

<ul style="list-style-type: none"> ● F-TF:B.5-7 <p>Model periodic phenomena with trigonometric functions</p> <p>5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.*</p> <p>6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.</p> <p>7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.*</p>	<p>MP.1 Make sense of problems and persevere in solving them.</p> <p>MP.2 Reason abstractly and quantitatively.</p> <p>MP.3 Construct viable arguments & critique the reasoning. of others.</p> <p>MP.4 Model with mathematics.</p> <p>MP.5 Use appropriate tools strategically.</p> <p>MP.6 Attend to precision.</p> <p>MP.7 Look for and make use of structure.</p> <p>MP.8 Look for and express regularity in repeated reasoning.</p>	<p>Concept(s):</p> <ul style="list-style-type: none"> ● Using trigonometric functions to model periodic phenomena. <p>Students are able to:</p> <ul style="list-style-type: none"> ● define transformation of trigonometric functions. ● identify the period, amplitude, frequency, phase shifts, and vertical translations of sinusoids. ● graph sinusoids. ● evaluate inverse trigonometric functions without a calculator. ● compose trigonometric functions using inverses. ● graph inverse trigonometric functions. <p>Learning Goal 3: Expand knowledge of transformations to include sinusoidal characteristics, and graph sinusoids.</p> <p>Learning Goal 4: Evaluate and graph inverse trigonometric functions.</p>
<ul style="list-style-type: none"> ● G-C:B.5 	<p>MP.1 Make sense of</p>	<p>Concept(s): Arc length and sector area</p>

<p>Find arc lengths and areas of sectors of circles</p> <p>5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.</p>	<p>problems and persevere in solving them.</p> <p>MP.2 Reason abstractly and quantitatively.</p> <p>MP.3 Construct viable arguments & critique the reasoning. of others.</p> <p>MP.4 Model with mathematics.</p> <p>MP.5 Use appropriate tools strategically.</p> <p>MP.6 Attend to precision.</p> <p>MP.7 Look for and make use of structure.</p> <p>MP.8 Look for and express regularity in repeated reasoning.</p>	<p>Students are able to:</p> <ul style="list-style-type: none"> • find arc length using both radian and degree measurements • find the central angle measurement of the sector in degrees and radian. <p style="text-align: center;">Learning Goal 5: Measure portions of a circle using arc length and sector area formulas in both radian and degree angles.</p>
<ul style="list-style-type: none"> • G-SRT: C <p>Define trigonometric ratios and solve problems involving right triangles</p>	<p>MP.1 Make sense of problems and persevere in solving them.</p> <p>MP.2 Reason abstractly and quantitatively.</p>	<p>Concept(s):</p> <ul style="list-style-type: none"> • Solving equations and problems using trigonometric ratios <p>Students are able to:</p> <ul style="list-style-type: none"> • solve problems involving the trigonometric functions of real numbers and the properties of trigonometric functions.

<p>6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles</p> <p>8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.</p>	<p>MP.3 Construct viable arguments & critique the reasoning. of others.</p> <p>MP.4 Model with mathematics.</p> <p>MP.5 Use appropriate tools strategically.</p> <p>MP.6 Attend to precision.</p> <p>MP.7 Look for and make use of structure.</p> <p>MP.8 Look for and express regularity in repeated reasoning.</p>	<ul style="list-style-type: none"> ● use trigonometric measurements to evaluate angle of elevation and angle of depression. ● calculate harmonic motion. <p style="text-align: center;">Learning Goal 6: Evaluate trigonometric functions and solve real-world applications involving directional bearings and harmonic motion.</p>
--	--	---

Formative Assessments	Summative Assessments
<ul style="list-style-type: none"> ● Student Whiteboards ● Checks for Understanding ● Teacher’s observation ● Desmos Activities ● Section Quizzes ● IXL 	<ul style="list-style-type: none"> ● Chapter Test ● Midterm ● Common Assessment ● Benchmark Assessment
Suggested Primary Resources	Suggested Supplemental Resources
PRECALCULUS - Functions & Graphs: 4th Edition TI-83/84 (College Prep)	IXL Desmos Activities

TI-89 (Honors) Desmos Graphing Calculator	Edpuzzle Quizlet Khan Academy Tutorials
Cross-Curricular Connections & 21st Century Skills	
<ul style="list-style-type: none"> ● Science, Technology, Engineering, and Mathematics (STEM) Literacy ● Critical Thinking & Problem Solving ● Communication and Collaboration ● Life and Career Skills 	
Essential Questions	Enduring Understanding
<ul style="list-style-type: none"> ● How can triangles relate to a circle? ● What are the major properties of the unit circle: it's values, symmetries, etc? ● How can trigonometric values be found using the unit circle? ● How can right triangle definitions be extended to apply to circular functions? ● What are the domain restrictions on inverse trig functions? ● How can the amplitude, period, and shifts of a sinusoid be found by analyzing a trigonometric function? ● How can sinusoids be used to model real-world phenomena like tides, the motion of a roller coaster, the motion of a piston, etc.? 	<ul style="list-style-type: none"> ● Trigonometry can be used not just to solve triangles, but to model waves. ● The important properties of waves, including amplitude, period, and shift, can be found by applying algebra to the trigonometric function. ● The unit circle is an important tool for finding trig values.

Differentiation & Real World Connections

504	<ul style="list-style-type: none"> ● preferential seating ● extended time on tests and assignments ● reduced homework or classwork ● verbal, visual, or technology aids 	<ul style="list-style-type: none"> ● modified textbooks or audio-video materials ● behavior management support ● adjusted class schedules or grading ● verbal testing
Enrichment	<ul style="list-style-type: none"> ● Utilize collaborative media tools ● Provide differentiated feedback ● Opportunities for reflection ● Opportunities for self-evaluation 	<ul style="list-style-type: none"> ● Encourage student voice and input ● Model close reading ● Distinguish long term and short term goals
IEP	<ul style="list-style-type: none"> ● Utilize “skeleton notes” where some required information is already filled in for the student ● Provide access to a variety of tools for responses ● Provide opportunities to build familiarity and to practice with multiple media tools ● Graphic organizers 	<ul style="list-style-type: none"> ● Leveled text and activities that adapt as students build skills ● Provide multiple means of action and expression ● Consider learning styles and interests ● Provide differentiated mentors
ELLs	<ul style="list-style-type: none"> ● Pre-teach new vocabulary and meaning of symbols ● Embed glossaries or definitions ● Provide translations ● Connect new vocabulary to background knowledge 	<ul style="list-style-type: none"> ● Provide flash cards ● Incorporate as many learning senses as possible ● Portray structure, relationships, and associations through concept webs ● Graphic organizers

At-risk	<ul style="list-style-type: none"> ● Purposeful seating ● Counselor involvement ● Parent involvement 	<ul style="list-style-type: none"> ● Contracts ● Alternate assessments ● Hands-on learning
21st Century Skills		
<ul style="list-style-type: none"> ● Creativity ● Innovation ● Critical Thinking 	<ul style="list-style-type: none"> ● Problem Solving ● Communication ● Collaboration 	
Integrating Technology		
<ul style="list-style-type: none"> ● Chromebooks ● Internet research ● Online programs 	<ul style="list-style-type: none"> ● Virtual collaboration and projects ● Presentations using presentation hardware and software 	
Career education		
<ul style="list-style-type: none"> ● Weekly Discussions: The value of mastering multiple languages in the workforce. 	<ul style="list-style-type: none"> ● Equity Discussions: People who benefit from knowing multiple languages. 	

Subject: Pre-Calculus	Grade: 11-12	Unit: 5 Analytic Trigonometry	4th Marking Period
Content Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills	
<ul style="list-style-type: none"> ● F-TF:B.5-7 <p>Model periodic phenomena with trigonometric functions</p> <p>5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.*</p>	<p>MP.1 Make sense of problems and persevere in solving them.</p> <p>MP.2 Reason abstractly and quantitatively.</p> <p>MP.3 Construct viable arguments & critique the reasoning. of others.</p> <p>MP.4 Model with mathematics.</p>	<p>Concept(s):</p> <ul style="list-style-type: none"> ● The fundamental trigonometric identities <p>Students are able to:</p> <ul style="list-style-type: none"> ● use inverse trigonometry to solve trigonometric equations ● use sum and difference formulas to evaluate trigonometric equations. ● use multiple-angle formulas to evaluate trigonometric functions. ● use half-angle formulas to evaluate trigonometric functions. 	

<p>6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.</p> <p>7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.*</p>	<p>MP.5 Use appropriate tools strategically.</p> <p>MP.6 Attend to precision.</p> <p>MP.7 Look for and make use of structure.</p> <p>MP.8 Look for and express regularity in repeated reasoning.</p>	<p>Learning Goal 1: Use inverse functions to solve trigonometric equations that arise in modeling contexts.</p>
--	--	---

<p>● F-TF:C.8-9</p> <p>Prove and apply trigonometric identities</p> <p>8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or</p>	<p>MP.1 Make sense of problems and persevere in solving them.</p> <p>MP.2 Reason abstractly and quantitatively.</p> <p>MP.3 Construct viable arguments & critique the reasoning of others.</p> <p>MP.4 Model with</p>	<p>Concept(s):</p> <ul style="list-style-type: none"> The fundamental trigonometric identities continued <p>Students are able to:</p> <ul style="list-style-type: none"> use the fundamental trigonometric identities to simplify trigonometric expressions. use the fundamental trigonometric identities to evaluate equations. use the fundamental trigonometric identities to prove expressions analytically.
--	---	--

<p>$\tan(\theta)$ and the quadrant of the angle.</p> <p>9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.</p>	<p>mathematics.</p> <p>MP.5 Use appropriate tools strategically.</p> <p>MP.6 Attend to precision.</p> <p>MP.7 Look for and make use of structure.</p> <p>MP.8 Look for and express regularity in repeated reasoning.</p>	<p>Learning Goal 2: Simplify trigonometric expressions using fundamental identities and general strategies.</p> <p>Learning Goal 3: Prove the validity of trigonometric expressions using the fundamental identities.</p>
---	--	---

Formative Assessments	Summative Assessments
<ul style="list-style-type: none"> ● Student Whiteboards ● Checks for Understanding ● Teacher's observation ● Desmos Activities ● Section Quizzes ● IXL 	<ul style="list-style-type: none"> ● Chapter Test ● Midterm ● Common Assessment ● Benchmark Assessment
Suggested Primary Resources	Suggested Supplemental Resources
<p>PRECALCULUS - Functions & Graphs: 4th Edition TI-83/84 (College Prep) TI-89 (Honors) Desmos Graphing Calculator</p>	<p>IXL Desmos Activities Edpuzzle Quizlet Khan Academy Tutorials</p>

Cross-Curricular Connections & 21st Century Skills	
<ul style="list-style-type: none"> ● Science, Technology, Engineering, and Mathematics (STEM) Literacy ● Critical Thinking & Problem Solving ● Communication and Collaboration ● Life and Career Skills 	
Essential Questions	Enduring Understanding
<ul style="list-style-type: none"> ● What are the basic trigonometric identities and how can they be used to simplify expressions? ● How can Basic Trigonometric Identities and Pythagorean Trigonometric Identities be used to simplify Trigonometric Equations? ● How do you rewrite trigonometric expressions in order to simplify and evaluate trigonometric functions? ● How do you verify a trigonometric identity? ● How do you solve trigonometric equations written in quadratic form or containing more than one angle? ● How do you simplify expressions and solve equations that contain sums or differences of angles? 	<ul style="list-style-type: none"> ● Use the fundamental trigonometric identities to evaluate trigonometric functions, simplify trigonometric expressions, and rewrite trigonometric expressions. ● Verify trigonometric identities. ● Use standard algebraic techniques to solve trigonometric equations. ● Solve trigonometric equations involving multiple angles. ● Use inverse trigonometric functions to solve trigonometric equations. ● Use sum and difference formulas to evaluate trigonometric functions, verify trigonometric identities and solve trigonometric equations. ● Use multiple-angle formulas to rewrite and evaluate trigonometric functions. ● Use half-angle formulas to rewrite and evaluate trigonometric functions.

Differentiation & Real World Connections

504	<ul style="list-style-type: none"> ● preferential seating ● extended time on tests and assignments ● reduced homework or classwork ● verbal, visual, or technology aids 	<ul style="list-style-type: none"> ● modified textbooks or audio-video materials ● behavior management support ● adjusted class schedules or grading ● verbal testing
Enrichment	<ul style="list-style-type: none"> ● Utilize collaborative media tools ● Provide differentiated feedback ● Opportunities for reflection ● Opportunities for self-evaluation 	<ul style="list-style-type: none"> ● Encourage student voice and input ● Model close reading ● Distinguish long term and short term goals
IEP	<ul style="list-style-type: none"> ● Utilize “skeleton notes” where some required information is already filled in for the student ● Provide access to a variety of tools for responses ● Provide opportunities to build familiarity and to practice with multiple media tools ● Graphic organizers 	<ul style="list-style-type: none"> ● Leveled text and activities that adapt as students build skills ● Provide multiple means of action and expression ● Consider learning styles and interests ● Provide differentiated mentors
ELLs	<ul style="list-style-type: none"> ● Pre-teach new vocabulary and meaning of symbols ● Embed glossaries or definitions ● Provide translations ● Connect new vocabulary to background knowledge 	<ul style="list-style-type: none"> ● Provide flash cards ● Incorporate as many learning senses as possible ● Portray structure, relationships, and associations through concept webs ● Graphic organizers
At-risk	<ul style="list-style-type: none"> ● Purposeful seating ● Counselor involvement ● Parent involvement 	<ul style="list-style-type: none"> ● Contracts ● Alternate assessments ● Hands-on learning

21st Century Skills	
<ul style="list-style-type: none"> ● Creativity ● Innovation ● Critical Thinking 	<ul style="list-style-type: none"> ● Problem Solving ● Communication ● Collaboration
Integrating Technology	
<ul style="list-style-type: none"> ● Chromebooks ● Internet research ● Online programs 	<ul style="list-style-type: none"> ● Virtual collaboration and projects ● Presentations using presentation hardware and software
Career education	
<ul style="list-style-type: none"> ● Weekly Discussions: The value of mastering multiple languages in the workforce. 	<ul style="list-style-type: none"> ● Equity Discussions: People who benefit from knowing multiple languages.

Subject: Pre-Calculus	Grade: 11-12	Unit: 6 Introduction to Calculus	4th Marking Period
Content Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills	

<ul style="list-style-type: none"> ● F-BF:A.1a,c <p>Build a function that models a relationship between two quantities</p> <p>1. Write a function that describes a relationship between two quantities.*</p> <p>a. Determine an explicit expression, a recursive process, or steps for calculation from a context.</p> <p>c. (+) Compose functions. <i>For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.</i></p>	<p>MP.1 Make sense of problems and persevere in solving them.</p> <p>MP.2 Reason abstractly and quantitatively.</p> <p>MP.3 Construct viable arguments & critique the reasoning. of others.</p> <p>MP.4 Model with mathematics.</p> <p>MP.5 Use appropriate tools strategically.</p> <p>MP.6 Attend to precision.</p> <p>MP.7 Look for and make use of structure.</p> <p>MP.8 Look for and express regularity in repeated reasoning.</p>	<p>Concept(s):</p> <ul style="list-style-type: none"> ● Limits and motion <p>Students are able to:</p> <ul style="list-style-type: none"> ● compose functions ● determine an explicit expression, a recursive process, and steps for calculation. <p>Learning Goal 1: Make a connection to function composition and end behavior models to evaluate the limit of functions.</p>
---	--	--

<ul style="list-style-type: none"> ● F-BF:B.4d <p>Build new functions from existing functions</p> <p>4. Find inverse functions.</p> <p>d. (+) Produce an invertible function from a non-invertible function by restricting the domain.</p>	<p>MP.1 Make sense of problems and persevere in solving them.</p> <p>MP.2 Reason abstractly and quantitatively.</p> <p>MP.3 Construct viable arguments & critique the reasoning. of others.</p> <p>MP.4 Model with mathematics.</p> <p>MP.5 Use appropriate tools strategically.</p> <p>MP.6 Attend to precision.</p> <p>MP.7 Look for and make use of structure.</p> <p>MP.8 Look for and express regularity in repeated reasoning.</p>	<p>Concept(s):</p> <ul style="list-style-type: none"> ● Inverse relations and inverse functions <p>Students are able to:</p> <ul style="list-style-type: none"> ● produce an invertible function from a non-invertible function by restricting its domain. ● use inverse functions to solve limits that arise in modeling contexts <p>Learning Goal 2: Use inverse function techniques to make a connection to limit expressions.</p>
---	--	--

<ul style="list-style-type: none"> ● F-TF:B.7 	<p>MP.1 Make sense of problems and persevere in</p>	<p>Concept(s):</p> <ul style="list-style-type: none"> ● Limits and continuity of inverse trigonometric functions
--	---	---

<p>Model periodic phenomena with trigonometric functions</p> <p>7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.*</p>	<p>solving them.</p> <p>MP.2 Reason abstractly and quantitatively.</p> <p>MP.3 Construct viable arguments & critique the reasoning. of others.</p> <p>MP.4 Model with mathematics.</p> <p>MP.5 Use appropriate tools strategically.</p> <p>MP.6 Attend to precision.</p> <p>MP.7 Look for and make use of structure.</p> <p>MP.8 Look for and express regularity in repeated reasoning.</p>	<p>Students are able to:</p> <ul style="list-style-type: none"> ● identify if a function is one-to-one on its domain. ● use inverse functions to solve trigonometric equations that arise in modeling contexts <p style="text-align: center;">Learning Goal 3: Evaluate the limit of inverse trigonometric functions and applications.</p>
--	---	--

Formative Assessments	Summative Assessments
<ul style="list-style-type: none"> ● Student Whiteboards ● Checks for Understanding ● Teacher’s observation ● Desmos Activities 	<ul style="list-style-type: none"> ● Chapter Test ● Midterm ● Common Assessment ● Benchmark Assessment

<ul style="list-style-type: none"> ● Section Quizzes ● IXL 	
Suggested Primary Resources	Suggested Supplemental Resources
PRECALCULUS - Functions & Graphs: 4th Edition TI-83/84 (College Prep) TI-89 (Honors) Desmos Graphing Calculator	IXL Desmos Activities Edpuzzle Quizlet Khan Academy Tutorials
Cross-Curricular Connections & 21st Century Skills	
<ul style="list-style-type: none"> ● Science, Technology, Engineering, and Mathematics (STEM) Literacy ● Critical Thinking & Problem Solving ● Communication and Collaboration ● Life and Career Skills 	
Essential Questions	Enduring Understanding
<ul style="list-style-type: none"> ● How do you find and interpret the limit of a function for a certain value of x? ● How do you evaluate limits that cannot be solved through use of direct substitution? ● How do you find the derivative of a function using differentiation rules? ● How do you find the limits of functions at infinity? 	<ul style="list-style-type: none"> ● Use the definition of a limit to determine whether limits of functions exist. ● Use properties of limits to evaluate limits. ● Use rationalization to evaluate limits of functions. ● Use graphing techniques to approximate the limit of a function. ● Define and use differentiation rules to evaluate the derivative of functions. ● Evaluate limits of functions at infinity.

Differentiation & Real World Connections

504	<ul style="list-style-type: none"> ● preferential seating ● extended time on tests and assignments ● reduced homework or classwork ● verbal, visual, or technology aids 	<ul style="list-style-type: none"> ● modified textbooks or audio-video materials ● behavior management support ● adjusted class schedules or grading ● verbal testing
Enrichment	<ul style="list-style-type: none"> ● Utilize collaborative media tools ● Provide differentiated feedback ● Opportunities for reflection ● Opportunities for self-evaluation 	<ul style="list-style-type: none"> ● Encourage student voice and input ● Model close reading ● Distinguish long term and short term goals
IEP	<ul style="list-style-type: none"> ● Utilize “skeleton notes” where some required information is already filled in for the student ● Provide access to a variety of tools for responses ● Provide opportunities to build familiarity and to practice with multiple media tools ● Graphic organizers 	<ul style="list-style-type: none"> ● Leveled text and activities that adapt as students build skills ● Provide multiple means of action and expression ● Consider learning styles and interests ● Provide differentiated mentors
ELLs	<ul style="list-style-type: none"> ● Pre-teach new vocabulary and meaning of symbols ● Embed glossaries or definitions ● Provide translations ● Connect new vocabulary to background knowledge 	<ul style="list-style-type: none"> ● Provide flash cards ● Incorporate as many learning senses as possible ● Portray structure, relationships, and associations through concept webs ● Graphic organizers

At-risk	<ul style="list-style-type: none"> ● Purposeful seating ● Counselor involvement ● Parent involvement 	<ul style="list-style-type: none"> ● Contracts ● Alternate assessments ● Hands-on learning
21st Century Skills		
<ul style="list-style-type: none"> ● Creativity ● Innovation ● Critical Thinking 	<ul style="list-style-type: none"> ● Problem Solving ● Communication ● Collaboration 	
Integrating Technology		
<ul style="list-style-type: none"> ● Chromebooks ● Internet research ● Online programs 	<ul style="list-style-type: none"> ● Virtual collaboration and projects ● Presentations using presentation hardware and software 	
Career education		
<ul style="list-style-type: none"> ● Weekly Discussions: The value of mastering multiple languages in the workforce. 	<ul style="list-style-type: none"> ● Equity Discussions: People who benefit from knowing multiple languages. 	

Appendix A

Audubon Public Schools

Engaging Students ~ Fostering Achievement ~ Cultivating 21st Century Global Skills

Written By: Erin Buthusiem, Patricia Martel, Ronald Latham
Reapproved June 2017

Course Title: Pre Calculus Unit Name: Prerequisites and Basic Functions
Grade Level: 11-12

Content Statements This unit reviews prior knowledge from Algebra 1 & 2 codifying it in standard terms with standard comparisons between functions.	NJSLS: A-APR.3; F-IF.4; F-IF.9; F-BF.4
Overarching Essential Questions How are functions the same? How are functions different?	Overarching Enduring Understandings All functions can be categorized and compared in terms of their domain, range, inverse, boundedness, extrema, continuity, graph, intercepts and asymptotes.

Unit Essential Questions

What are the 10 basic functions?

How can the domain and range of a function be found graphically? Using algebra?

What is asymptotic behavior, and how can asymptotes be found graphically and algebraically?

How can algebraic tools like factoring, distributing, and simplifying rational expressions be used to analyze the behavior of functions?

What is end behavior, and how is it related to the limit at infinity?

How can coefficients and constants be used to translate, stretch, and rotate the graphs of functions?

What are the limitations of graphing calculators, and what kind of functions often yield misleading results when solved technologically?

How can curve-fitting technology be used to convert data into mathematical models, which can then be used to extrapolate future results?

How can we verify by composition that one function is the inverse of another?

Unit Enduring Understandings

There are 10 major "families" of functions. Each family has its own shape when graphed, and each family has its own restrictions and limits.

Unit Rationale

This unit is necessary to codify prior knowledge from previous courses using the vocabulary and criteria that will be used for future in depth study of these concepts

Unit Overview

The analysis of Functions using algebra and graphing calculator tools is a core skill for the study of higher math. Each type of function has unique attributes, including domain and range restrictions, asymptotes, and discontinuities, and these attributes are essential to understanding calculus.

Resources

PRECALCULUS-Functions & Graphs: 4th Edition (ISBN 0-201-61136-8)

Kuta Software

Fluid Math

TI SMART Software

Tutorials on www.brightstorm.com

TI-83 or TI-84 Graphing Calculators.

Suggested Student Activities

Sketch basic functions and label the important points and properties

Match graphs of functions to their algebraic equations, with and without calculator assistance

Given a basic function, describe verbally and graphically how changing the values in the function affects the graph

Use curve-fitting technology to find equations given tables of data values

Zoom in and out on the graphs of functions using graphing calculators, identifying misleading or hard to see attributes of graphs

Given a data table, select the basic function that best models the data.

Identify the horizontal and vertical asymptotes of a rational function algebraically and graphically

Key Terms

Boundedness: if a function has a horizontal growth or decay

Concavity: if a function has a curve, describing it in terms of its direction

Continuity: if a function can be written with one smooth motion or is composed of several parts

Degree: highest power of exponent

Domain: all possible values of x

Range: all possible values of y

Periodicity

Audubon Public Schools

Engaging Students ~ Fostering Achievement ~ Cultivating 21st Century Global Skills

Written By: Erin Buthusiem, Patricia Martel, Ronald Latham

Reapproved June 2017

Course Title: Pre Calculus

Unit Name: Linear, Quadratic, Power, Polynomial & Rational Functions

Grade Level: 11-12

<p>Content Statements This unit further explores polynomial and power functions in terms of their end behaviors, intercepts, and methods of solution and simplification.</p>	<p>NJSLS: F-IF.7:A–D; F-IF.8:A–B; F-IF.9; F-BF.3; N-RN.1 – 3; N-CN.4 – 6</p>
<p>Overarching Essential Questions How do we find solutions of polynomial, power, and rational functions? How do the end behaviors relate to each function? How are solutions, zeros, and x-intercepts related? Why are the sum or product of two rational numbers rational; the sum of a rational number and an irrational number irrational; and the product of a nonzero rational number and an irrational number irrational?</p>	<p>Overarching Enduring Understandings End behaviors are controlled by the individual function and determine the limits of each function as it approaches infinity or negative infinity. Solutions to polynomial and power functions can be found graphically, using the quadratic formula, factoring, long division, synthetic division and substitution, and graphing calculator functions. Solutions to equations show up as x intercepts on a graph and zeros in synthetic division.</p>

Unit Essential Questions

How can a linear function be determined given two points?

How can rate of change be determined given two values of a function, and how is rate of change related to slope?

How can a quadratic function be converted from standard to vertex form, and what is the significance of each?

When is an appropriate to use "r" and "r-squared" analysis to determine the correct model of a data set, and what are the limits of these statistics?

What do the graphs of functions with negative or fractional exponents look like?

How can the roots, shape, and end behavior of a polynomial function be determined by looking at its algebraic equation?

Why is the graphing calculator a poor tool for analyzing polynomial functions?

How can long division of polynomials be used to find intercepts, asymptotes, and the general behavior of rational functions?

How can graphing rational functions identify zeros and asymptotes when suitable factorizations are available, and show end behavior?

How can representing complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers)?

Explain why the rectangular and polar forms of a given complex number represent the same number.

Unit Enduring Understandings

Linear, quadratic, power, polynomial, and rational functions each have unique shapes and properties.

Algebraic techniques can be used to find intercepts, slopes of lines, the vertex of a parabola, or the end behavior and roots of a polynomial function.

How can calculating the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints?

Unit Rationale

This unit is essential to codify and extend the methods of solution and relate them to each other and the graphs of the function.

Unit Overview

Modeling real-world phenomena using algebraic functions, which can be simplified and analyzed with or without graphing technology, is a major application of higher algebra and calculus.

Resources

PRECALCULUS-Functions & Graphs: 4th Edition (ISBN 0-201-61136-8)

Kuta Software

Fluid Math

TI SMART Software

Tutorials on www.brightstorm.com

TI-83 or TI-84 Graphing Calculators.

Suggested Student Activities

Finding the function for a line given two points

Converting a quadratic function into vertex form and graphing the resulting parabola

Determining the average rate of change for linear or quadratic functions; drawing conclusions based on the rate of change procedure

Modeling quadratic and power functions using curve-fitting technology

Rewriting radical and rational functions in power function form

Hand-sketching polynomial functions

Graphing polynomial functions on the calculator, recognizing the misleading elements and limitations of the calculator graphs\

Hand-sketching rational functions, identifying roots and asymptotes

Key Terms

Conjugate- binomial term of similar structure to original with opposite sign between the terms used to foil out an imaginary term and rationalize a denominator of a radical function

Power Function- a monomial function

Zero- a value for x then when plugged into synthetic division results in a remainder of zero

Audubon Public Schools

Engaging Students ~ Fostering Achievement ~ Cultivating 21st Century Global Skills

Written By: Erin Buthusiem, Patricia Martel, Ronald Latham

Reapproved June 2017

Course Title: Pre Calculus

Unit Name: Exponential & Logarithmic Functions

Grade Level: 11-12

Content Statements This unit covers identifying, dissecting, describing, graphing and manipulating exponential, logarithmic, logistic, natural log and natural base equations.	NJSLS: F-IF.7.E; F-IF.8:A–B; F-BF.1:B–C; F-BF.5
Overarching Essential Questions What is a logarithm, ln, e and a logistical function? How are they used?	Overarching Enduring Understandings Logarithms are another form exponential function that can be solved following its own set of rules. Ln and e are inverse exponential functions of each other following their own set of rules for solution.

Unit Essential Questions

How are exponential and logarithmic functions related?

What are the shapes and properties of exponential and logarithmic functions, and how can they be transformed?

How can populations, annuities, and radioactive half lives be modeled exponentially?

How can the properties of exponential and logarithmic functions be used to manipulate and simplify problems?

What are orders of magnitude and how are they related to exponential functions?

How can $A = Pert$ and other formulas be used to model exponential growth and decay?

How can the graphs of exponential and logarithmic functions show intercepts and end behavior?

Unit Enduring Understandings

Logarithms can be solved by rewriting them as exponents, and vice-versa. Exponential formulas

Like: $A = Pert$ can be used to solve growth problems in science and business. The number "e" is used to model continuous growth and decay.

Unit Rationale

This unit gives direct application for exponential, logarithmic and logistical functions to prepare students for further units of study as well as methods of solution for their chemistry course work.

Unit Overview

Exponential functions are used to model a variety of growth and decay problems that are important in higher math and the sciences. Logarithmic functions are the inverse of exponential functions and can be used to solve exponential functions or model slow growth or decay, which are major scientific, business and engineering concepts.

Resources

PRECALCULUS-Functions & Graphs: 4th Edition (ISBN 0-201-61136-8)

Kuta Software

Fluid Math

TI SMART Software

Tutorials on www.brightstorm.com

TI-83 or TI-84 Graphing Calculators.

Suggested Student Activities

Rewriting logarithmic functions as exponential functions and vice versa

Finding logarithms with and without a calculator

Modeling growth and decay using exponential functions

Deriving the e constant

Exploring the logarithmic nature of pH and the Richter's Scale

Using Newton's Law of Cooling to calculate temperature change.

Build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

Compose functions to model temperature in the atmosphere as a function of height, and the height of a weather balloon as a function of time, then the temperature at the location of the weather balloon as a function of time.

Key Terms

Logarithmic- alternate form to exponential form to model and solve exponential functions

Logistic- having a limit to growth being divided by the exponential function creating the growth

Audubon Public Schools

Engaging Students ~ Fostering Achievement ~ Cultivating 21st Century Global Skills

Written By: Erin Buthusiem, Patricia Martel, Ronald Latham

Reapproved June 2017

Course Title: Pre Calculus

Unit Name: Trigonometry and the Unit Circle

Grade Level: 11-12

<p>Content Statements This unit relates the unit circle to right triangles and trigonometry as well as the relationships between the six trigonometric ratios, their reciprocals and their inverses.</p>	<p>NJSLS: F-IF.7.E; F-TF.1–9</p>
<p>Overarching Essential Questions How can triangles relate to a circle? How are trigonometric functions related to anything in the real world? How can Basic Trigonometric Identities and Pythagorean Trigonometric Identities be used to simplify Trigonometric Equations?</p>	<p>Overarching Enduring Understandings Coordinates along the circumference of a circle are cyclical and this cycle can be modeled using the triangles formed between the center of the circle and the coordinates of any point on the circle. Like trigonometric functions and coordinates on a circle, the phases of the moon, tides, and seasons are all cyclical. More concretely, the position of a piston in an engine, Ferris wheel, bike tire, and perpetual motion machine all function cyclically and positions within their cycles can be found using trig functions.</p>

Unit Essential Questions

What are the major properties of the unit circle: it's values, symmetries, etc?

How can trigonometric values be found using the unit circle?

How can right triangle definitions be extended to apply to circular functions?

What are the domain restrictions on inverse trig functions?

How can the amplitude, period, and shifts of a sinusoid be found by analyzing a trigonometric function?

How can sinusoids be used to model real-world phenomena like tides, the motion of a roller coaster, the motion of a piston, etc.?

What are the basic trigonometric identities and how can they be used to simplify expressions?

What is the ambiguous case for an SSA triangle?

How can the Law of Sines and Law of Cosines be used to solve non-right triangles? How can vector notation be used to solve problems involving flight, forces, and trajectories?

How can the graphs of trigonometric functions showing period, midline, and amplitude?

Unit Rationale

Trigonometric functions and their applications can be and are used in a wide variety of areas. A thorough understanding of these concepts is necessary to be successful in future study of mathematics. They are essential to solve many HSPA and SAT II problems as well as a staple concept in many college placement tests.

Unit Enduring Understandings

Trigonometry can be used not just to solve triangles, but to model waves. The important properties of waves, including amplitude, period, and shift, can be found by applying algebra to the trigonometric function. The unit circle is an important tool for finding trig values.

Unit Overview

The unit circle, which is used to find the values of trig and inverse trig functions, is a vital tool in higher math study. The properties of sinusoids can be used to model a variety of real world phenomena, and it's important to a conceptual understanding of higher math to be able to

sketch trigonometric functions with and without technology.

Resources

PRECALCULUS-Functions & Graphs: 4th Edition (ISBN 0-201-61136-8)

Kuta Software

Fluid Math

TI SMART Software

Tutorials on www.brightstorm.com

TI-83 or TI-84 Graphing Calculators.

Suggested Student Activities

Solving right triangles

Creating reference angles to convert circular functions into right triangles

Sketching the unit circle and entering values

Finding values of circular functions on the unit circle

Hand sketching sinusoids given a trigonometric function

Creating trig functions and using them to model data, given data points or a sketch.

Simplifying trigonometric identities

Using sum and difference identities to find the trig values of angles

Applying the law of sines and cosines to non-right triangles

Determining whether an SSA case describes 0, 1 or 2 triangles

Key Terms

Amplitude- the difference between the median and its extrema in sine and cosine, change in amplitude in music changes volume

Frequency- the number of periods occurring within the standard period, change in frequency in sound waves determines pitch

Inverse or Arc- the function that is applied to undo or peel back the given trig function, a method of solution

Period- length of piece being repeated to generate trig graph

Reciprocal- multiplication inverse, can cancel out a function through multiplication

Shift- the amount of horizontal and vertical transformation of a trig function from its parent function

Audubon Public Schools

Engaging Students ~ Fostering Achievement ~ Cultivating 21st Century Global Skills

Written By: Erin Buthusiem, Patricia Martel, Ronald Latham

Reapproved June 2017

Course Title: Pre Calculus

Unit Name: Vectors, Parametric Equations, and Polar Equations

Grade Level: 11-12

Content Statements

This unit introduces vectors in the plane, performing vector operations, and using vectors to represent quantities such as force and velocity. It also introduces parametric equations to simulate motion and polar coordinates to represent points in the coordinate plane.

NJSLS:

N-VM.1–5

<p>Overarching Essential Questions</p> <p>How can vector operations be used to find direction angles and magnitude, calculate velocity, and find a force?</p> <p>How can we find a parametric equation for a line segment, given endpoints?</p> <p>How can we relate polar and rectangular form?</p>	<p>Overarching Enduring Understandings</p> <p>The basic vector operations such as addition, dot product, and magnitude are used to solve vector applications. Using coordinate conversion equations, we can relate rectangular and polar equations.</p>
<p>Unit Essential Questions</p> <p>How can we calculate the effect of wind velocity?</p> <p>How can magnitude and dot product be used to find angles between vectors?</p> <p>How can dot product be used to find a projection of one vector onto another?</p> <p>How can magnitude and dot product be used to find a force?</p> <p>Given parameters, how can we graph a set of parametric equations?</p> <p>How can objects in motion be modeled using parametric equations?</p> <p>What are polar coordinates and how can they be used to simplify circular functions?</p> <p>How can we find distances using polar coordinates?</p>	<p>Unit Enduring Understandings</p> <p>Vectors and vector operations can be used to find angles between vectors, projection, and velocity.</p> <p>Parametric equations can be used to simulate motion.</p> <p>Polar equations can be converted to rectangular form and rectangular equations can be converted to polar form.</p>
<p>Unit Rationale</p> <p>This unit is essential to the study of vectors, which are used extensively in physics, engineering, and applied mathematics, and the study of parametric and polar equations to analyze distance and motion.</p>	<p>Unit Overview</p> <p>Vectors and vector operations as component vectors relate to parameters in parametric equations. Applications of parametric equations simulate motion.</p>

Resources

PRECALCULUS-Functions & Graphs: 4th Edition (ISBN 0-201-61136-8)

Kuta Software

Fluid Math

TI SMART Software

Tutorials on www.brightstorm.com

TI-83 or TI-84 Graphing Calculators.

Suggested Student Activities

Writing vectors in component form

Represent scalar multiplication graphically by scaling vectors and reversing their direction

Perform scalar multiplication component-wise

Solve problems involving velocity and other quantities that can be represented by vectors.

Writing equations in parametric form

Converting to and from polar coordinates

Key Terms

Magnitude- length of a vector

Dot Product- used to find angles between vectors, projection, and force.

Parametrization- a set of parametric equations for a curve

Polar coordinates- a method of graphing with a spherical point of reference, used when a grid approach is not possible

Audubon Public Schools

Engaging Students ~ Fostering Achievement ~ Cultivating 21st Century Global Skills

Written By: Erin Buthusiem, Patricia Martel, Ronald Latham

Reapproved June 2017

Course Title: Pre Calculus

Unit Name: Analytic Geometry- Conic Sections

Grade Level: 11-12

<p>Content Statements Students will understand the geometric principles that are present in these conic section equations and graphs.</p>	<p>NJSLS: G-GPE.1–3</p>
<p>Overarching Essential Questions What is a conic section? How can I construct an equation for a conic section?</p>	<p>Overarching Enduring Understandings A conic section is a shape formed by slicing a series of cones at various angles.</p>
<p>Unit Essential Questions How do I write the equation of a parabola? How can I construct a graph of a parabola, given a standard form equation? Is a circle also an ellipse? How can I construct a graph of an ellipse, given a standard form equation? How do I write the equation of an ellipse? How do I write the equation of a hyperbola? How can I find the vertices and foci of a hyperbola? How do I write polar equations for conics?</p>	<p>Unit Enduring Understandings Foci have a large impact on the shape of an ellipse, hyperbola, and parabola. These shapes are used in technology, medicine, and electronics.</p>
<p>Unit Rationale These geometric relationships are used in many different fields. They are used in CGI animation, to focus the beams of light in flashlights and headlights, and planets travel in elliptical paths.</p>	<p>Unit Overview This unit emphasizes how conic section equations can be used to model shapes and pictures.</p>

Resources

PRECALCULUS-Functions & Graphs: 4th Edition (ISBN 0-201-61136-8)

Kuta Software

Fluid Math

TI SMART Software

Tutorials on www.brightstorm.com

TI-83 or TI-84 Graphing Calculators.

Suggested Student Activities

Create a design or image using two of each conic equations, color it, and submit an equation key that can be used to recreate.

Key Terms

Ellipse- A regular oval shape, traced by a point moving in a plane so that the sum of its distances from two other points (the foci) is constant.

Hyperbola- A symmetrical open curve formed by the intersection of a cone with a plane at a smaller angle with its axis than the side of the cone.

Parabola- A symmetrical open plane curve formed by the intersection of a cone with a plane parallel to its side.

Audubon Public Schools

Engaging Students ~ Fostering Achievement ~ Cultivating 21st Century Global Skills

Written By: Erin Buthusiem, Patricia Martel, Ronald Latham

Reapproved June 2017

Course Title: Pre Calculus

Unit Name: Probability, Combinatorics, & Discrete Math

Grade Level: 11-12

<p>Content Statements This unit reviews concepts necessary to solve probability, statistical, counting vector problems.</p>	<p>NJSLS: S-ID.6: A–C; S-MD.1–7; A-APR.5; A-SSE.4</p>
<p>Overarching Essential Questions What methods can I use to solve probability problems? How can I use counting methods and drawing methods to solve problems? How can I solve statistical and data based problems?</p>	<p>Overarching Enduring Understandings Many methods including factorials, permutations, and combinations can be used to solve probability problems. Counting and mapping outcomes are reliable methods to determine total possible outcomes and min required amounts to solve the handshake problem, path problems, and figure drawing problems. Statistical and data based problems can be solved with mean, median, mode, bar graphs, deviation models, histograms, box whisker plots and frequency tables.</p>

Unit Essential Questions

How can the counting principle be used to find sample spaces and probabilities?

What is the difference between a permutation and combination?

What is the nature of the combination function, and how does it relate to binomial expansion?

How can the coefficients of an expanded binomial be found?

What techniques can be used to calculate the probabilities of compound events?

How can series of numbers be expressed in sequential or summation notation?

How can summation formulas be used to find the sum of a large series of numbers?

How can technology be used to easily express data in histograms and box-and-whisker plots?

What is a normal distribution?

How can standard deviation and the 68-95-99.7 rule be used to classify and organize data?

How can the formula for the sum of a finite geometric series be used to solve problems?

How can we define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space?

Unit Enduring Understandings

The Binomial theorem can be used to expand polynomials and to determine the probability of an event. The Binomial Theorem can be applied easily using tools like Pascal's Triangle and the combination function. Tools like standard deviation are used to determine whether data falls into the "average", "high average", or "low average" range.

Unit Rationale

This unit is a review of previous concepts in these areas to prepare students for taking the HSPA, the SAT and SAT II, and college math placement exams.

Unit Overview

Basic concepts like the Counting Principal can be expanded into combinatorics and the binomial theorem, which can be used to determine the outcomes of compound events. Data sets that fall into a normal distribution can be analyzed using statistical concepts like standard deviation. These are critical concepts in the study of statistics, a branch of mathematics with important economic, scientific, and sociological applications.

Resources

PRECALCULUS-Functions & Graphs: 4th Edition (ISBN 0-201-61136-8)

Kuta Software

Fluid Math

TI SMART Software

Tutorials on www.brightstorm.com

TI-83 or TI-84 Graphing Calculators.

Suggested Student Activities

Finding the number of possible passwords, security codes, routes between points, and so on using the counting principal.

Analyzing Pascal's Triangle for its values and symmetries.

Determining the relationships among Pascal's Triangle, the combination function, and the coefficients of an expanded polynomial.

Expanding an nth degree polynomial.

Using the coefficients of an expanded polynomial to determine the probability that an event happened by chances.

Finding the outcomes and probabilities when two dice are rolled, cards are drawn from a standard deck, 10 coins are flipped.

Rewriting algebraic equations in sequential notation and vice versa.

Expanding series written in Summation notation and writing series in summation notation.

Adding large sums of numbers using summation formulas.

Using the graphing functions on the TI-84 to create histograms and box-and-whisker graphs

Identifying outliers using the Inter-quartile range formula.

Finding standard deviation using TI-84 functions

Graphing normal distribution/bell curve functions and analyzing then using the 68-95-99.7 rule.

Evaluate and compare strategies on the basis of expected values; compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable chances of having a minor or a major accident.

Key Terms

Deviation- a statistical model of how spaced out data points are from the mean, determines reliability of the data for predictive purposes

Network- a model composed of a lines and vertices representing points of intersection

Audubon Public Schools

Engaging Students ~ Fostering Achievement ~ Cultivating 21st Century Global Skills

Written By: Erin Buthusiem, Patricia Martel, Ronald Latham

Reapproved June 2017

Course Title: Pre Calculus Unit Name: Intro to Calculus
Grade Level: 11-12

<p>Content Statements</p> <p>This unit relates end behavior models to limits of functions. An extensive study of limits at a point and at infinity will follow. Limits will then be used to develop the meaning of a derivative at a point and at infinity.</p>	<p>NJSLS:</p> <p>A-APR.3; F-IF.7:A–D; F-IF.8:A–B; F-BF.1:B–C; F-IF.7:A–D;</p>
<p>Overarching Essential Questions</p> <p>What is a limit?</p> <p>How does a limit relate to functions?</p> <p>How can limits be used in real world applications?</p>	<p>Overarching Enduring Understandings</p> <p>A limit is the value of a function as in approaches and given point or infinity. Limits are used to replace “missing” points or to approximate values for prediction and planning purposes</p>

Unit Essential Questions

What is the meaning of the limit of a function?
What is the difference between instantaneous velocity and average velocity?
What is the physical significance of the average rate of change of a function?
What is the graphical and conceptual meaning of the derivative?
How can limits be used to find the derivative of a function?
What are limits at infinity and how are they related to horizontal asymptotes?
How can the area between a line or a simple curve and the x-axis be found algebraically and geometrically?
What is the graphical and conceptual meaning of the integral of a function?
How can the power rule be used to simplify the process of finding the derivative?

Unit Enduring Understandings

The derivative is the instantaneous rate of change of a function, it is related to the formula for slope, and it can be found using algebraic methods. The integral is the area beneath a curve between two points and there are geometric methods for finding integrals.

Unit Rationale

This unit sums up the year of study by showing how all of the functions studied can be uniformly analyzed in terms of their limits and prepares students for the study of calculus and its applications.

Unit Overview

It is essential to understand that it is possible to find the instantaneous rate of change in a function given only one point on the function's graph using limits and derivatives. The ability to find the exact area between a curve and the x-axis on a given interval using integrals is the core topic of calculus and the gateway to the study of higher mathematics.

Resources

PRECALCULUS-Functions & Graphs: 4th Edition (ISBN 0-201-61136-8)

Kuta Software

Fluid Math

TI SMART Software

Tutorials on www.brightstorm.com

TI-83 or TI-84 Graphing Calculators.

Suggested Student Activities

Finding the average rate of change in a function using slopes.

Interpreting the meaning of a limit given a graph

Reducing rational expressions to find limits.

Interpreting discontinuous graphs to find left and right-handed limits.

Determining the difference between a function's limit and its value on a graph

Finding the derivative at a point using the formal definition of the derivative.

Finding the equation for a function's derivative given the formal definition.

Graphing lines and half-circles, then finding the area under those graphs on given regions

Finding the area under other curves using shifting rules and geometric concepts.

Using derivative and integral functions on the calculator to analyze curves and find values.

Applying the Power Rule to find the slopes of tangent lines and equations for tangent lines.

Key Terms

Limit- the value of a function, or y value, at a given value of x or as x approaches either infinity

Derivative- how the y value changes as the x value changes

Appendix**Differentiation**

Enrichment	<ul style="list-style-type: none"> ● Utilize collaborative media tools ● Provide differentiated feedback ● Opportunities for reflection ● Encourage student voice and input ● Model close reading ● Distinguish long term and short term goals
Intervention & Modification	<ul style="list-style-type: none"> ● Utilize “skeleton notes” where some required information is already filled in for the student ● Provide access to a variety of tools for responses ● Provide opportunities to build familiarity and to practice with multiple media tools ● Leveled text and activities that adapt as students build skills ● Provide multiple means of action and expression ● Consider learning styles and interests ● Provide differentiated mentors ● Graphic organizers
ELLs	<ul style="list-style-type: none"> ● Pre-teach new vocabulary and meaning of symbols ● Embed glossaries or definitions ● Provide translations ● Connect new vocabulary to background knowledge ● Provide flash cards ● Incorporate as many learning senses as possible ● Portray structure, relationships, and associations through concept webs ● Graphic organizers
21st Century Skills	

- Creativity
- Innovation
- Critical Thinking
- Problem Solving
- Communication
- Collaboration

Integrating Technology

- Chromebooks
- Internet research
- Online programs
- Virtual collaboration and projects
- Presentations using presentation hardware and software